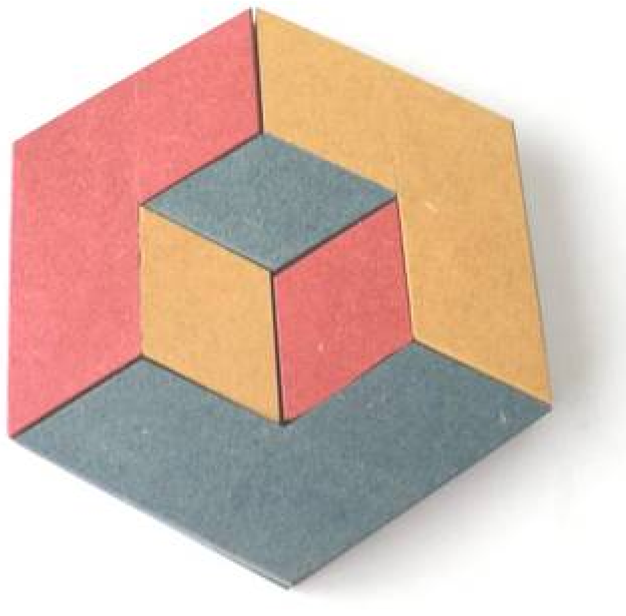
Albert Gübeli



Visualization of Euclidian Four-Dimensional Space

I wish to thank Hansjörg Illi for suggestions and corrections of the content and Trudi Richner for corrections of the English text.

Visualization of Euclidian Four-Dimensional Space by

Albert Gübeli

Contents

Introduction

In flatland (1) the world was described in the plane with its problems. But with our spatial perception these problems suddenly move away.

In the literature (2-4) there are always hints given as to how our limited spatial perception could be, from the view of higher beings.

Under the concept of dimension I understand the original meaning of *Dimensio*, spatial measurements in different (or all possible) directions.

The 3-dimensional space

Figure 1 left, shows the well known 3-dimension space, Figure 1 right, shows the whole space in 6 directions *x*, *-x*, *y*, *-y*, *z*, *-z* which defines eight 3-dimensional cells. The diagonal opposite cells have no common faces.

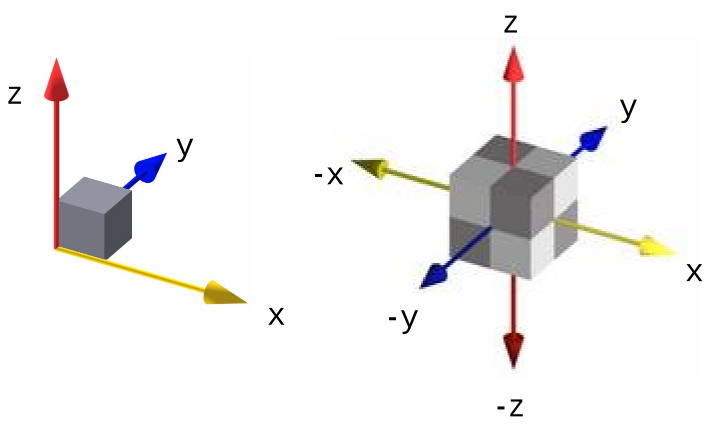


Figure 1

Figure 2 shows two objects (half of one cube) in two 3-dimensional cells, one with only positive directions and the other with the same values but negative directions. The objects are not identical, but by surprise chiral (enantiomorphs). I name them left and right handed. One left and one right handed half part can not be put together to form one cube.

Conclusion, with 6 directions, the whole space around one centre splits in to 4 right handed and 4 left handed cells. The whole space cannot be described with 8 right or 8 left handed cells.

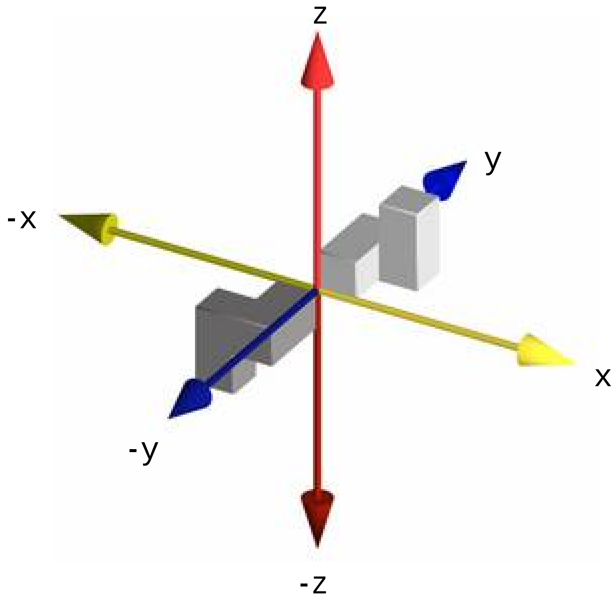


Figure 2

The 4-dimensional space

With figures I would like to introduce you to the 4-dimensional space with angles between the directions from 109.47122°. Figure 3 shows how the angles are created by shear (1 unit) of the deformable square (3x3 units).

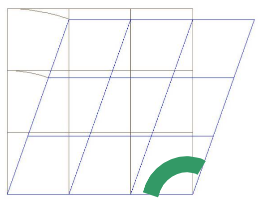
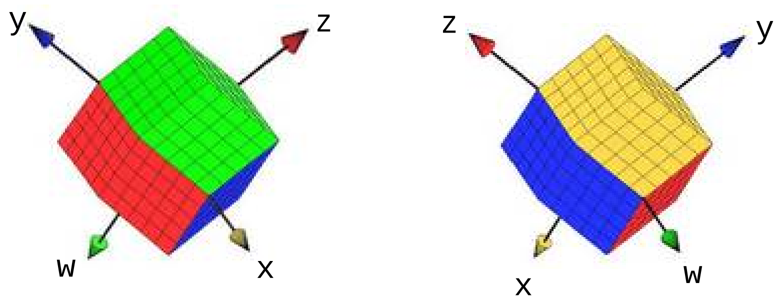


Figure 3

Figure 4 shows the representation of the 4-dimensional space from front and back with 4 coordinates, each with 5 units. Four 3-dimensional rhombohedron cells, green, red, blue and yellow build the 4-dimensional space, the Rhombic Dodecahedron (5, 7).



View front

View back

Figure 4

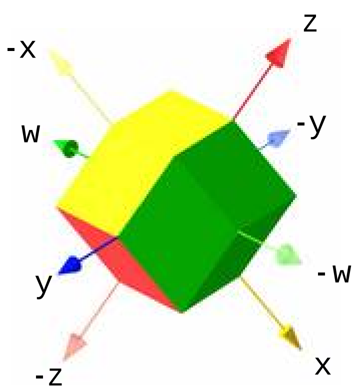
The red rhombohedron cell is defined by the directions *x*, *y*, *w*, the green cell by *x*, *y*, *z*, the yellow cell by *y*, *z*, *w* and the blue cell by *x*, *z*, *w*.

Each of the four 3-dimensional rhombohedron cells has one plane common with the other three without any chirality.

The whole space around the centre can be described with four directions (coordinates); the chirality does not play any role.

We will draw now the four Rhombohedron cells with negative units which are opposite and chiral to the cells with positive units.

Figure 6 shows the eight 3-dimensional rhombohedron cells on the left the four with positive units and on the right the four with negative units, depending on the definition.



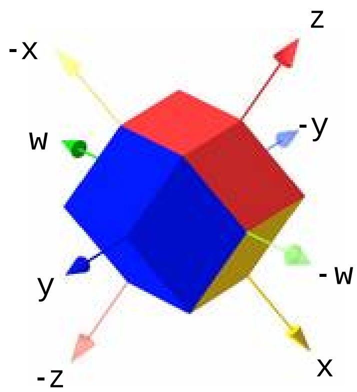


Figure 6

The four opposite 3-dimensional Rhombohedron cells with negative units have no common plane with the four 3-dimensional Rhombohedron cells with positive units and form together two identical Rhombic Dodecahedrons, one with positive units and one with negative units.

In the two Rhombic Dodecahedrons with positive and negative units, the chirality of the parts change.

Figure 7 shows one left handed part (yellow) in the 3-dimensional Rhombohedron *w*, *y*, *z* with positive units and one right handed part with the same units but negative -*w*, -*y*, -*z* (yellow transparent).

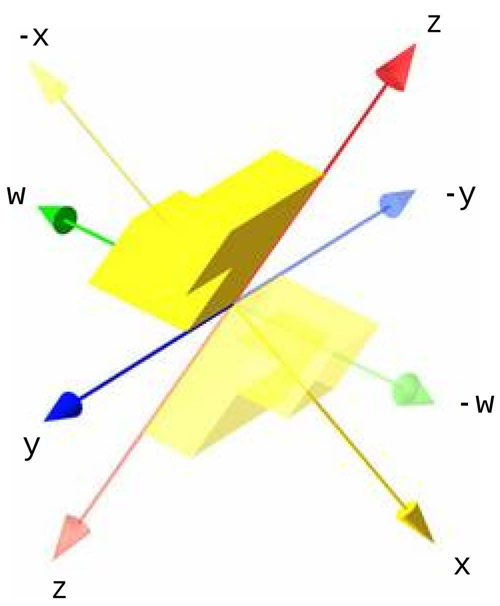


Figure 7

But contrary to the 3-dimensional space the 4-dimensional space defines the whole space around the centre, one with positive units and one with negative units; in other words, the 4-dimensional space is homogeneous.

It follows, that the 4 dimensional spaces is defined by two chirality Rhombic Dodecahedrons of the same value.

Figure 8 shows the positive 4-dimensional space with 5 units per coordinate.

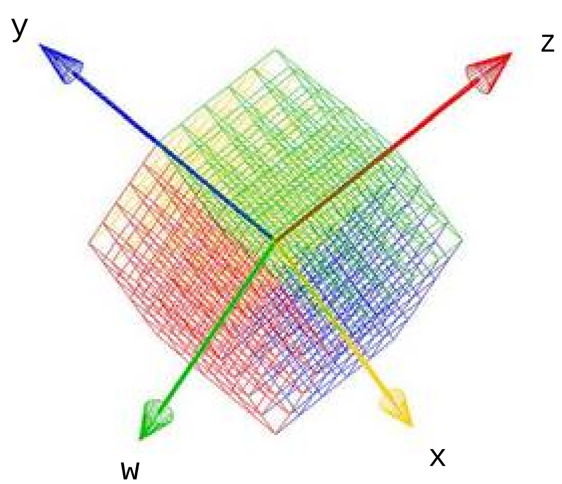


Figure 8

We are looking at the 4-dimensional graph with 5 units in each direction from different angles. At first we are looking in the direction from one coordinate *w* as example, see Figure 9, and we get the well known Isometric graph.

First conclusion: For the illustration of unlimited plane you need at least 3 directions.

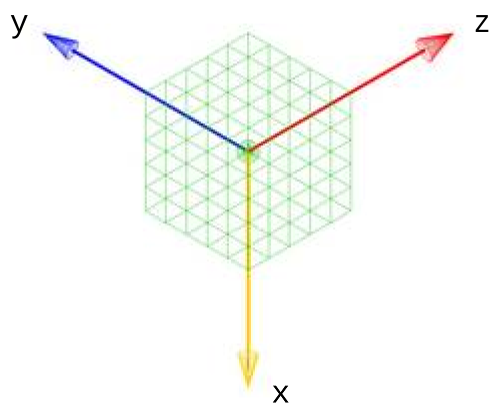


Figure 9

We are looking at the 4-dimensional graph with 5 units in each direction from a bisector of two directions as example x/w or y/z (see Figure 10) and we get the well known coordinate graph.

Second conclusion: For the illustration of unlimited space (without negative units) you need at least four directions.

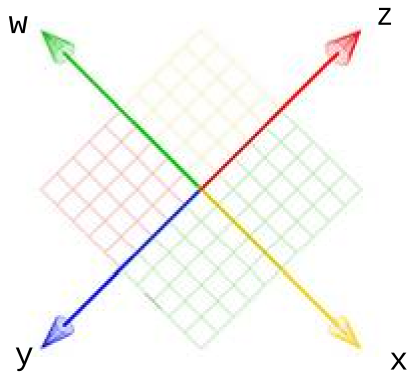
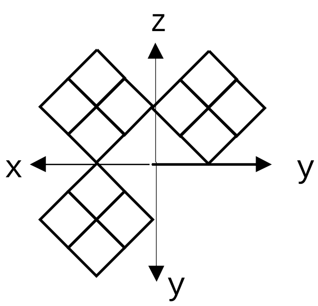


Figure 10

Exercise: Views from

front right



and top

How looks the convex 3-dimensional body
with the above views?

It could not be a big surprise, that as solution we find the Rhombic Dodecahedron. Figure 11 represents with three squares (light grey) the 3-dimensional space as we have learned.

The projected shadows (grey) in the direction of the 3 coordinates x, y, z shows the Rhombic Dodecahedron from front, right and top.

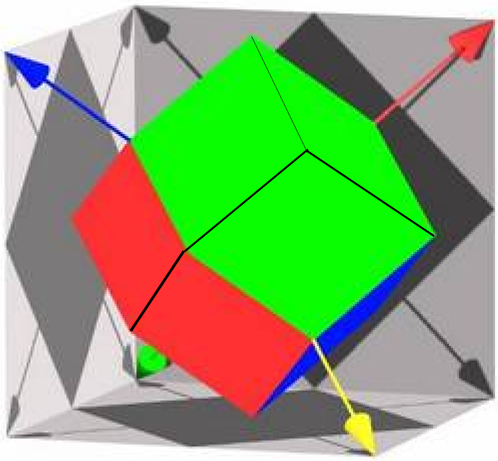


Figure 11

Now we have worked out all the necessary background for explaining my experience with the development of simultaneously forced mechanism for my puzzles.

Development of puzzles with simultaneously forced mechanism

In my work, I have taken the habitats to move for the development of simultaneously forced mechanism between plane, 3-dimensional space and the 4-dimensional space.

This led me to interesting conclusions. In the plane it needs just 3 identically parts to create one simultaneously forced mechanism.

Figure 12 shows one carried out from unlimited possibilities. A set of variations is presented by the following puzzle HEX LÉON (8).

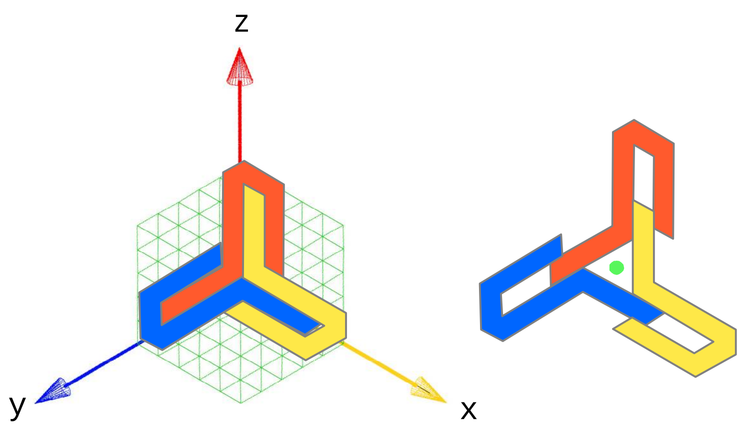


Figure 12

In the coordinate graph the linear simultaneously forced mechanism exits only in two directions (6), see Figure 13.

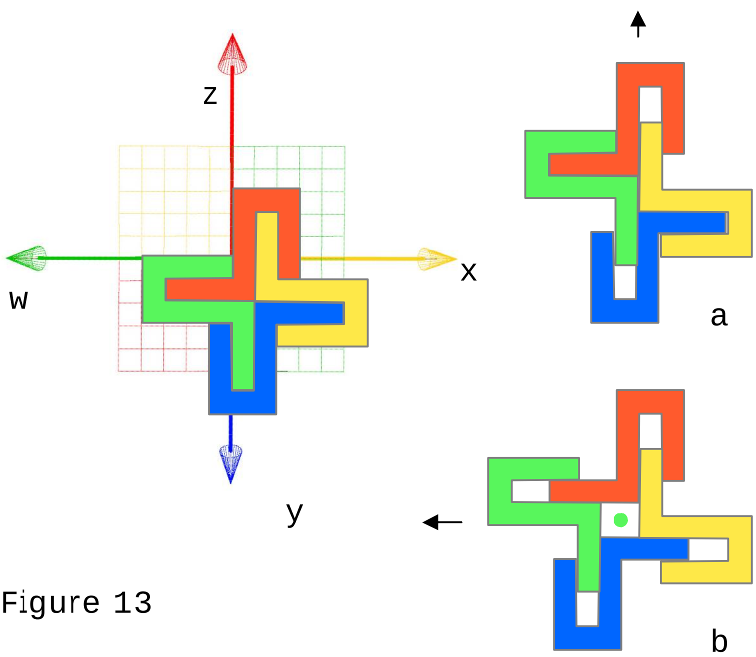


Figure 13

I have specially drawn the mechanism on the representation of 4-dimensional graph with 5 units, Figure 9.

Now there happens a big surprise. If you transform this mechanism in to the 4-dimensional space (Figure 13), the mechanism is functioning with just

two left- and two right handed identical parts.

The left handed part fits into the right handed part and vice versa.

It does not exist with four identical parts a simultaneously forced mechanism in space.

This I will show you with the help of my puzzle RHOMBO LÉON (9), a possibly carry out of unlimited variations.

My conclusions

You can turn it how you like, there only exist simultaneously forced mechanism with the following qualities:

In a line	2 identical pieces
In a plane	3 identical pieces right- or left handed
In a space	4 pieces, two left- and two right- handed identical pieces

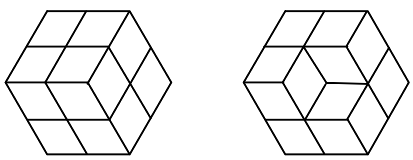
Until now I have not found a simultaneously forced mechanism with more then four pieces in the space, also not in systems with more than four directions.

The number of directions of a simultaneously forced mechanism determines the dimension in accordance with our definition and as conclusion; the space has to be 4-dimensional.

HEX LÉON (8)

The play object HEX-LÈON consists of three identical parts. One part is built from two pieces with 4 Rhombuses on two levels. The mechanism has for base the combinations of two pieces in two levels and it distinguishes that the three parts must been moved simultaneously inwards or outwards. Sequential put togethers or take aways are not possible.

The following arrangements of the 12 Rhombuses in one level are possible:



The cover picture and Figure 14 shows two variations of carry outs.

Figure 15 shows all the pieces in their positions that can be combined to create parts. From all the possible combinations of parts there are probably 64 which function as simultaneously forced mechanism.

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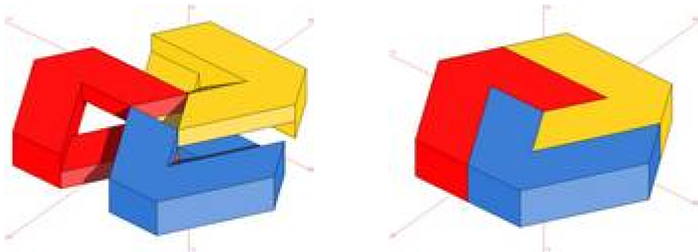


Figure 14

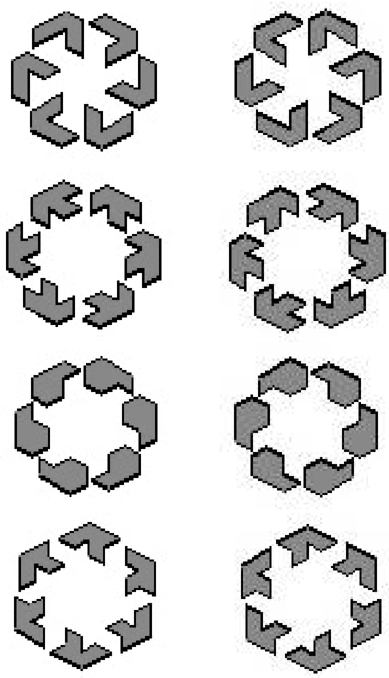


Figure 15

RHOMBO LÉON (9)

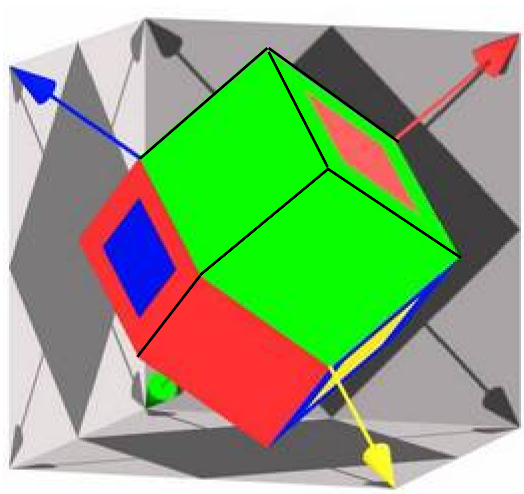


Figure 16

The Puzzle Rhombo Léon consists of 2 right handed (green, blue) and 2 left handed (red, yellow) pieces in four-dimensional space.

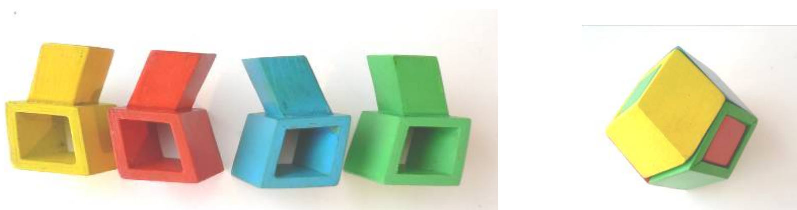


Figure 17

Figure 18: Views from

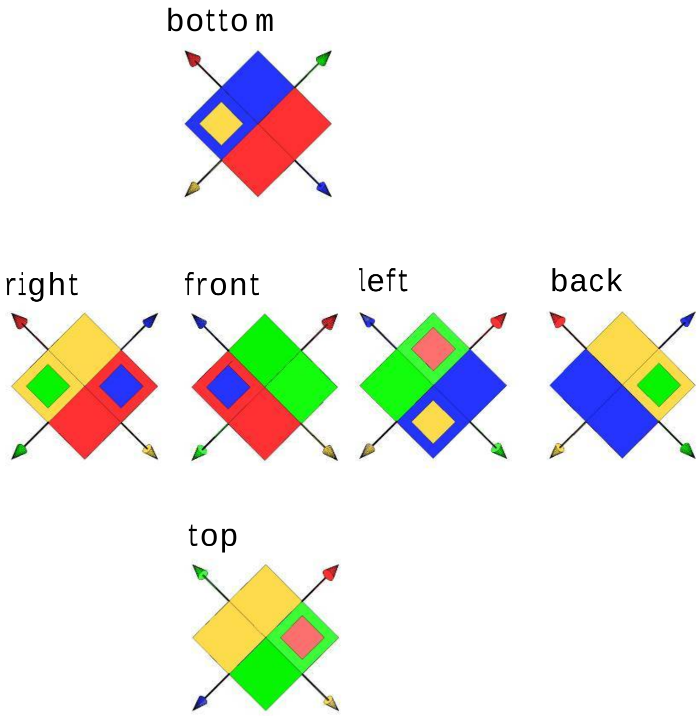
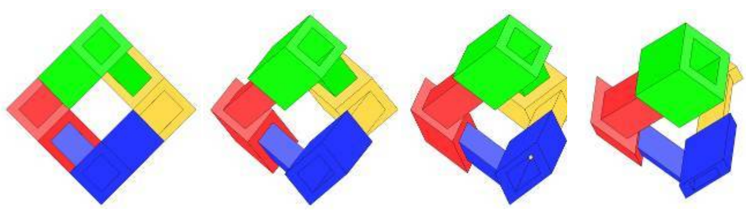


Figure 19: Puzzle Rhombo Léon open
View left, then step by step turned



(1) Flatland: A Romance of Many Dimensions, Edwin A. Abbott

(2) The Fourth Dimension, Rudy Rucker

(3) Speculations on the Fourth Dimension, Selected Writings of Charles H. Hinton, Edited by Rudolf v.B. Rucker, Dover Publicatioons

(4) Beyond the third dimension, Thomas F. Banchoff, Scientific American Library

(5) Die Rhomboeder-Bausteine des Albert Gübeli, Christoph Pöppe, Redakteur bei Spektrum der Wissenschaft

(6) Würfeleien, Albert Gübeli, albinegri Schriftenreihe Nr.4d

(7) Vom Würfel zum Rhombododekaeder, Albert Gübeli und Georges Wick, albinegri Schriftenreihe Nr.1d

(8) Puzzle HEX LÉON, Buyer's source http://www.shapeways.com/shops/albinegri

(9) Puzzle RHOMBO LÉON, Buyer's source http://www.shapeways.com/shops/albinegri